

Longitudinal hydrodynamics from event-by-event Landau initial conditions

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(Dated: April 1, 2014)

We investigate three-dimensional ideal hydrodynamic evolution, with Landau initial conditions, incorporating event-by-event variation with many events and transverse density inhomogeneities. We show that the transition to boost-invariant flow occurs too late for realistic setups, with corrections of $\mathcal{O}(20 - 30\%)$ expected at freezeout for most scenarios. Moreover, the deviation from boost-invariance is correlated with both transverse flow and elliptic flow, with the more highly transversely flowing regions also showing the most violation of boost invariance. We conclude that if longitudinal flow is not fully developed at the early stages of heavy ion collisions, 2+1 dimensional hydrodynamics is inadequate to extract transport coefficients of the quark-gluon plasma.

PACS numbers: 25.75.-q, 25.75.Dw, 25.75.Nq

The quantitative modeling of matter produced in high energy heavy ion collisions with relativistic hydrodynamics is now a well-established field, following the widely cited announcement that matter produced at the relativistic heavy ion collider (RHIC), behaves as a “perfect fluid” [1–6]. The evidence for this behavior comes from the successful modeling of RHIC anisotropic flow by boost-invariant hydrodynamics [7–12]. It is now clear that the same fluid-like behavior persists at the LHC [13–15].

It is commonly argued that, given precise enough data on soft physics, chiefly momentum spectra and their azimuthal anisotropy, the transport coefficients of matter created in ultrarelativistic heavy ion collisions can be quantitatively constrained. Several research groups are moving in this direction [16–21].

These models are all based on the reduction, either exact or approximate, of the problem to a 2+1 dimensional system [22], based on the symmetry of boost-invariance. Essentially, the system is assumed to have *as an initial condition* a longitudinal flow that is Hubble-like in the beam direction (usually associated with the “z” coordinate) *only*. This means that, initially,

$$v_z = \frac{z}{t}, \quad y_s = y_f = \langle y \rangle_p, \quad (1)$$

where y_s and y_f are respectively the spacetime and flow rapidities

$$y_s = \frac{1}{2} \ln \left(\frac{t+z}{t-z} \right), \quad y_f = \frac{1}{2} \ln \left(\frac{1+v_z}{1-v_z} \right),$$

$$\langle y \rangle_p = \frac{1}{2} \left\langle \ln \left(\frac{E+p_z}{E-p_z} \right) \right\rangle \simeq \frac{1}{2} \ln \cot \frac{p_z}{|p|}$$

with y_p being usually referred to as pseudo-rapidity. A further simplification comes from assuming that all initial dynamics does not depend on y

$$\frac{dN}{dy} = 0, \quad \frac{dv_T}{dy} = 0 \quad (2)$$

or, equivalently not on t, z separately, but just on $\tau = \sqrt{t^2 - z^2}$ (evolved from an initial time τ_0) and transverse degrees of freedom. An initial condition that respects Eq. (1) but not Eq. (2) will slowly degrade the constraints of Eq. (1), as shown in [23–25].

Initially, a different model has originally been advocated as the obvious initial state for the hydrodynamic evolution of the fluid: Landau hydrodynamics [26–29]. In this picture, the energy that forms the bulk of the expanding fireball “stops” at midrapidity at time zero (in the lab frame). The initial distribution of matter is therefore a “pancake”, of thickness Δ related to the boosted charge radius R of the nuclei with nucleon mass m_N at center of mass energy of $\sqrt{s_{NN}}$ where

$$\Delta \rightarrow \Delta_{lim} \simeq \frac{R}{\gamma} = \frac{Rm_N}{(\sqrt{s_{NN}}/2)}. \quad (3)$$

In a more general implementation, Δ need not be defined by Eq. (3) and can be a free parameter, reflecting the spread in configuration space of low x gluons. The initial Landau condition is defined by the assumption that the initial “pancake” has no existing longitudinal flow at all, unless there are initial inhomogeneities which lead to a net momentum in local transverse space. (This is known as the “firestreak model” [30, 31]). Boost invariance is badly broken at the beginning of the fireball evolution and such a pancake has very little in common with the scenario used in [22]. Indeed, the interplay between longitudinal and transverse effects could be highly non-trivial [24]. Physically, one can consider Bjorken and Landau as two extremes: In the Bjorken scenario, the nuclei originally pass through each other with minimal reinteraction and strings that stretch between colliding gluons arise in parallel to other strings. In the Landau scenario, they “stick together” or at least leave some energy in the middle.

While computationally reducing one dimension is highly desirable, a physics justification would be needed for the approximation of Eq. (1). A direct measure-

ment of dN/dy is inconclusive. On the one hand, it fits a Gaussian well at all energies, with universal limiting fragmentation, as expected in [26–28]; Moreover, strong violations of boost-invariance considerably lessen the HBT puzzle [32]. However, the *multiplicity* dependence on $\sqrt{s_{NN}}$ is not exactly that predicted in [26–28]. This by itself does not rule out the Landau scenario, as it can be accounted for by treating the initial thickness evolution with $\sqrt{s_{NN}}$ as a free parameter, as done in the Bjorken scenario.

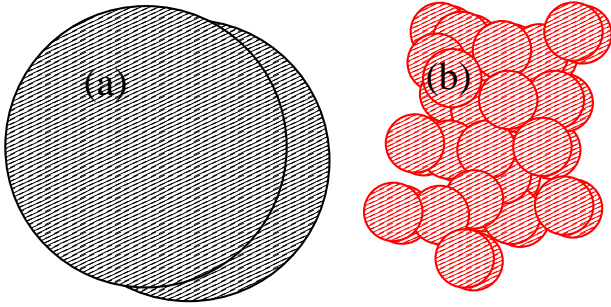


FIG. 1. A homogeneous “pancake” Landau initial condition (panel (a)), and actual Glauber initial conditions (panel (b)) for a typical event.

There are two main arguments one can give for the Bjorken limit being more appropriate: The first one is that the perturbative partonic picture of the system [33, 34] makes this initial condition natural. However, even in the weakly-coupled limit, low x partons could lead to a breakdown of Eq. (2) (see for example, [35, 36]). Moreover, if the initial state is *strongly coupled* from the beginning, one could indeed expect that it would appear much more Landau-like [37, 38] than Bjorken-like; the interaction strength at the beginning is currently a very controversial topic.

The second reason is that, for mid-rapidity data, it is widely suggested that the distinction between Bjorken and Landau evolution is irrelevant. As is clear from [26], Landau evolution converges to Bjorken evolution after some sufficient time. The reason for this behavior is that longitudinal flow forms on the scale of $\sim \Delta/c_s$ while transverse flow forms on a much larger scale $\sim R/c_s$ where c_s is the speed of sound. Hence, since $\Delta \ll R$ for $\sqrt{s_{NN}} \gg 1$ GeV, initially the system can be considered, as indeed it is in [26], to be a purely 1D expanding “sharp step.” As again shown in [26, 28, 29], the long-term longitudinal evolution of such a system at mid-rapidity is indistinguishable from that of [22]. Hence, 2+1 dimensional hydrodynamics can be safely used even if, at the very initial stage [24], the system is very far from boost invariance. Landau evolution at mid-rapidity can be treated as Bjorken with $\tau_0 \sim 1 \text{ fm} \times \text{GeV}/\sqrt{s_{NN}}$. Perhaps, this scaling will give unrealistically low initial times at the LHC, but since 2+1 simulations are only weakly sensitive to time [18], this might not be a fatal issue. This idea, however, has two flaws: First of all, for a non-central

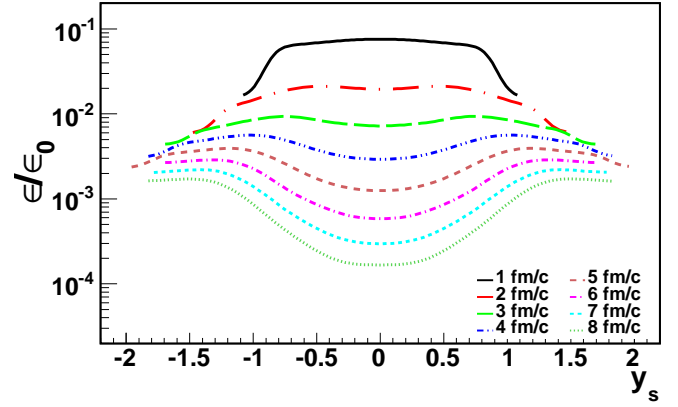


FIG. 2. (color online) Ratio of energy density at the indicated time to the initial energy density as a function of rapidity.

collision, where anisotropic flow is most expected, locality and longitudinal momentum conservation imply that the system develops an additional initial longitudinal momentum imbalance, with extra longitudinal momentum due to the local (in transverse space) imbalance between the target and projectile $\rho_{part}^{P,T}(x_T) = d^2 N_{part}^{P,T}/dx_T^2$ transverse participant density. This means that locally the initial $\gamma_z v_z$ is, for a Δ given by Eq. (3),

$$\frac{v_z(x_T)}{\sqrt{1-v_z^2(x_T)}} = \frac{\rho_{part}^P(x_T) - \rho_{part}^T(x_T)}{\rho_{part}^T(x_T) + \rho_{part}^P(x_T)} K \quad (4)$$

where $K = \sqrt{s_{NN}}/m_N$ when $\Delta = \Delta_{lim}$ in Eq. (3), while in general it is reasonably assumed to be related to Δ by a rescaling

$$K \simeq \frac{\sqrt{s_{NN}}}{m_N} \frac{\Delta_{lim}}{\Delta} = \frac{2R}{\Delta}. \quad (5)$$

This initial flow is trivially not boost invariant and it is not clear it disappears at *any* finite time for a general system evolving from a Landau initial condition.

Additionally, the “Landau→Bjorken” reasoning assumes that the longitudinal timescale is much larger than the transverse one. This is certainly true if the transverse scale is given as a radius of a homogeneous “pancake” of radius R given by an *average* of many events as in Fig. 1 (a). It is however less clear that such a hierarchy holds for a *typical* event as in Fig. 1(b). The inclusion of subnucleonic strong QCD fields [20, 39] make this hierarchy even more dubious as the events with the strongest anisotropic coefficients would also have the most prominent “hotspots.” Potentially, this effect makes the boost-invariant picture irrelevant *even for late-time hydrodynamics*: The more homogeneous regions will be more similar in their longitudinal expansion to [22], while the more inhomogeneous regions would, on their own, evolve to a *3D Hubble* expansion [40]. The interplay between regions of different symmetry, and local instabilities [40, 41]

makes any symmetry dubious. And, the fact that initial state inhomogeneities are indeed large even at high energies can be regarded as experimentally well established by measurements of odd coefficients of harmonic flow [42, 43].

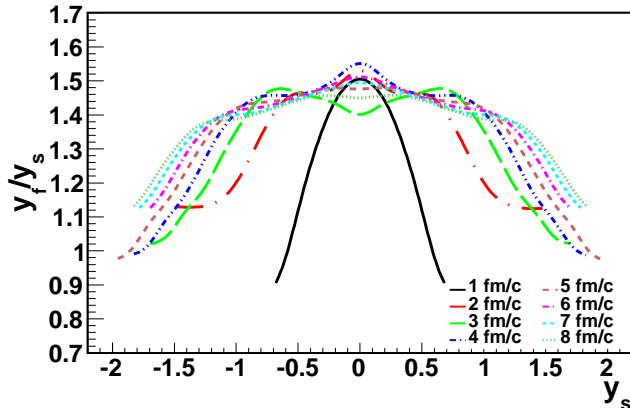


FIG. 3. (color online) The y_f/y_s ratio as a function of rapidity at several times for the hydrodynamic expansion, averaged over all events.

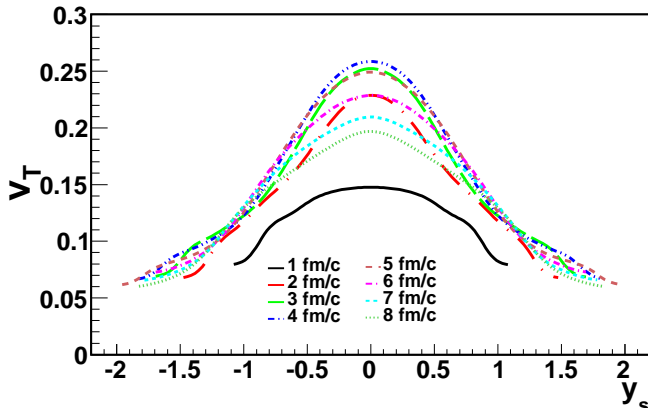


FIG. 4. (color online) Transverse velocity as a function of rapidity at several times for the hydrodynamic expansion, averaged over all events.

To investigate these effects further, one needs to perform 3+1D calculations starting from Landau initial conditions and transverse inhomogeneities. In this work, we use an event-by-event Glauber model to generate initial-state transverse energy distributions, with the longitudinal density distribution being given by a Landau profile.

The Glauber Monte Carlo description of two colliding Au^{197} nuclei at 200 GeV was used to generate the initial condition relevant to RHIC. Nucleons were distributed as per a Wood-Saxon distribution with radius 6.38 fm, and diffuseness 0.535 fm. The impact parameters were simulated randomly following a distribution of $d\sigma/db =$

$2\pi b$. The nucleons were assumed to have no hard-core and the condition for nucleon-nucleon collision is that the inter-nucleon distance d should satisfy $\pi d^2 < \sigma_{NN}$, where $\sigma_{NN} = 42$ mb is the nucleon-nucleon cross section.

We then use the CL-SHASTA code developed in [44] to evolve this configuration according to ideal hydrodynamics, $\partial_\mu T^{\mu\nu} = 0$ with

$$T_{\mu\nu} = (e + p)u_\mu u_\nu + pg_{\mu\nu} \quad (6)$$

$$u_\mu = \cosh(y_L) (1, v_T \sin(\theta), v_T \cos(\theta), \sinh(y_L)) \quad (7)$$

with an ideal gas equation of state, $p = e/3$, $c_s = 1/\sqrt{3}$, and $\Delta = 0.1$ fm, and longitudinal flow given by Eq. (4). Our results do not qualitatively change if the longitudinal thickness is changed by $\mathcal{O}(50 - 100\%)$.

The Titan supercomputer at the Oak Ridge National Laboratory Leadership Computing Facility is used to collect an ensemble of these numerically intensive calculations which is large enough to infer event-by-event correlations. For relativistic hydrodynamical calculations, the 3+1 Sharp and Smooth Transport Algorithm (SHASTA) was recently completely rewritten using the OpenCL computational framework to work on accelerators like Graphical Processing Units (GPUs) [44]. Parallelized algorithm kernels written in OpenCL run on GPUs with concurrent execution of thousands of streams. For this Letter adjustments were made for optimal use of the powerful NVIDIA GPUs and batch system of the Titan supercomputer. Using redesigned algorithms and harnessing the processing power of GPUs, the hydrodynamical calculations have a speedup of a factor of $\sim 100\times$ for a given node, scaled to large numbers of Titan nodes. This allowed us to accumulate a large ensemble of event-by-event statistics with unprecedented efficiency for relativistic hydrodynamical simulations.

In order to organize the hydrodynamic expansion into thousands of execution streams, the problem is reduced by *domain decomposition*. This leads to a grid structure in the spatial dimensions where the grid elements are still connected but can be processed separately. The grid size depends on hardware and algorithm type. The present implementation of the grid includes 8 million grid cells which covers ± 10 fm in each spatial dimension. Each grid cell holds the physical properties in that spatial region and one kernel per physical quantity is used to modify them accordingly throughout the expansion.

After simulating 2000 events for a given configuration and initial conditions, each evolving the millions of grid cells over 300 small time steps in the lab frame which covers an expansion until 10 fm/c, we divide them into spacetime rapidity slices. The energy density evolution as a function of spacetime rapidity is shown in Fig. 2, which follows the trend in [26] to $\sim 20\%$ precision, as expected from correction due to transverse and elliptic flow.

We then calculate the longitudinal flow rapidity y_f as well as the transverse flow on each slice of rapidity, averaged over the entire transverse volume, to explore boost

invariance. Fig. 3 shows the ratio y_f/y_s as a function of y_s at various times of the evolution. If the system were exactly boost invariant, y_f/y_s would be strictly unity. As expected, the ratio does evolve towards unity as time progresses and the system cools; however, it would be a gross oversimplification to treat the ratio as a constant or unity, even at significantly later times. At freezeout, we predict y_f/y_s to be above unity by about 20% around midrapidity. At earlier stages, relevant for the formation of transverse and elliptic flow ($t \sim \epsilon R/c_s$), these corrections are of order 50%.

The reason these corrections are larger than generally anticipated is revealed in Fig. 4 which shows that the transverse velocity as a function of the spacetime rapidity significantly violates Eq. (2). For these initial conditions, the transverse velocity increases until 4 fm/c of the evolution and then eventually decreases at late times.

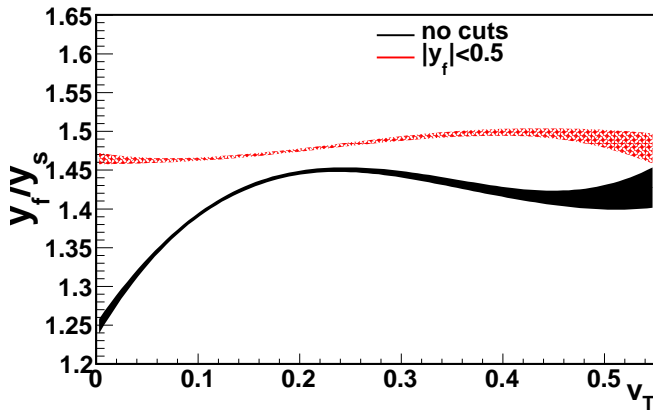


FIG. 5. (color online) Ratio y_f/y_s as a function of mean transverse velocity for $|y_f| < 0.5$ (red) and with no restriction on y_f (black).

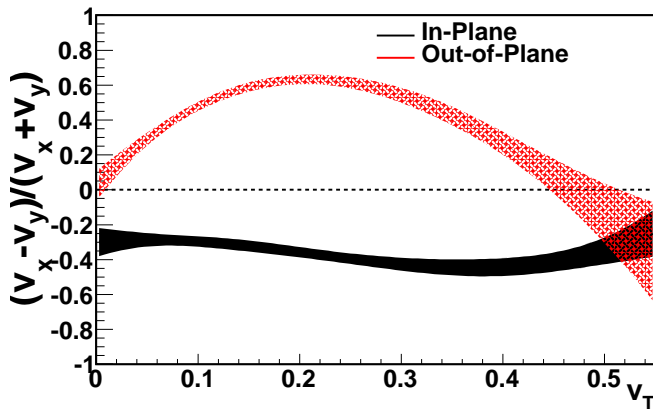


FIG. 6. (color online) Distribution of transverse flow anisotropy versus mean total transverse velocity for in-plane (black) and out-of-plane (red) flow.

The relevance of this dynamics for transverse degrees of freedom is further elucidated in Fig. 5, which shows the dependence of y_f/y_s , an indicator of the degree of violation of boost invariance, on transverse flow. Thus, if the Landau initial condition is more appropriate, transverse flow and its azimuthal anisotropies form, to a certain extent, in strongly non-boost invariant regions. This is readily understood, as such regions are precisely the places where transverse gradients are larger w.r.t. longitudinal ones. Hotspots can also have a non-zero longitudinal momentum and vorticity [46] (the “firestreak”), further invalidating local boost-invariance. As Fig. 5 however shows, this result somewhat depends on the rapidity region being explored. A restriction in flow rapidity, approximately tracking the pseudorapidity, will ensure v_T is independent of the degree of boost invariance. Such a cut, however, does nothing to make the evolution examined more boost-invariant, since y_f/y_s remains very well away from unity.

Fig. 6 shows the anisotropy of the in-plane and out of plane flow, as a function of transverse flow. The combination of the results of Figs. 4, 5, and 6 indicates that dynamics relevant for transverse and anisotropic flow significantly violates boost invariance if Landau initial conditions are assumed.

The main shortcoming of this analysis is that the hydrodynamics was assumed to be ideal. However, it should be noted that viscosity is precisely sensitive to differences between y_s and y_f examined here. In standard 2+1 dimensional hydrodynamics, the effect of viscosity is to lengthen the evolution of the system by transforming longitudinal gradients into a slower cooling rate of the system. Turning on a higher longitudinal flow than that expected by boost invariance should slow cooling by as much as 30% with respect to the estimates given in this work. While the effect of such terms will have to be explored in full 3+1 viscous hydrodynamic simulations, a Landau initial condition should be compatible with a considerably higher viscosity than that assumed in 2+1 models.

The viability of the computations performed here depends, of course, on the longitudinal structure of the event really being close to the Landau limit. We do not at the moment know whether this is the case, as it depends crucially on one of the most controversial issues in our field, whether the system is weakly coupled (which generally implies a Bjorken type evolution) or strongly coupled in the initial, thermalizing stages of the system’s evolution. As there is presently insufficient theoretical control to answer this question, this study motivates a specific test of boost invariance which should be experimentally feasible. Figs. 5 and 6 suggest that an event-by-event study of $\langle p_T \rangle$ and v_2 as functions of rapidity could yield a correlation between transverse flow and rapidity characteristic of strong non-boost invariant longitudinal flow. A natural observable to consider here is the one proposed to study viscosity in the boost-invariant hypothesis in [45]. A non-trivial monotonic centrality and

rapidity dependence for this quantity could signal the fluctuations described here. In addition, the vorticity polarization measurements suggested in [46] could also be promising to explore, given the dependence of *local* angular momentum on Eq. (4).

In conclusion, we have shown that, provided the system is Landau-like in its initial stages, it will not, as commonly expected, evolve to a Bjorken-like stage within realistic timescales. Furthermore, the deviation from boost invariance is directly correlated with the development of

transverse and elliptic flow, the characteristic signatures used to demonstrate and quantitatively study the hydrodynamics of the quark-gluon plasma. In view of these results, the transport properties of the medium created in heavy ion collisions could be considerably different from those usually assumed.

This research used resources of the Oak Ridge Leadership Computing Facility at the Oak Ridge National Laboratory, which is supported by the Office of Science of the U.S. Department of Energy under Contract No. DE-AC05-00OR22725.

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